

TG 496
May 1963
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IN
STATISTICAL INFERENCE
ASSOCIATED WITH
SATELLITE TRACKING**

by

William H. Guier

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ABSTRACT

SOME SPECIAL PROBLEMS IN STATISTICAL INFERENCE ASSOCIATED WITH SATELLITE TRACKING^{*}

William H. Guier⁺

Satellite tracking systems (notably optical and doppler) have been developed to such a degree that the accuracy of orbit determination is no longer limited by the accuracy of the tracking data itself but is almost wholly limited by other parameters associated with the satellite motion and tracking system. The principal parameters limiting this accuracy are those associated with the earth's gravitation force field and the locations of the satellite tracking stations. Significant improvement in the values of a few of these parameters has already been made via studies of satellite motion but, in most cases, they have been made without the use of statistical inference techniques. However, further significant improvement in the values of all of the parameters affecting tracking accuracy will require special applications of the methods of statistical inference.

The Navy's doppler tracking system provides ample high quality data for statistically inferring very accurate values of these parameters. However, employing inference techniques to tracking data obtained from such a system gives rise to some unusual problems. In particular the validity of inference techniques depends very strongly on correctly estimating the

^{*}This work was supported by the Department of the Navy, Bureau of Weapons, under Contract NOW 62-0604-c.

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character of and correlations in the experimental tracking data. This paper will discuss briefly some of the more difficult problems associated with specifying the errors in the data, the consequent problems in determining optimum statistical weighting factors, and finally the difficulties in realistically estimating the probable errors in the inferred values for the parameters.

SOME SPECIAL PROBLEMS IN STATISTICAL INFERENCE ASSOCIATED WITH SATELLITE TRACKING

In the few years that have followed the launching of the first artificial Earth satellite, satellite tracking systems have developed to such an extent that the quality of the tracking data is no longer the principal limiting factor to tracking accuracy. In particular, several systems have been shown to provide high quality tracking data and over short intervals of time (order of one day) have the ability to determine satellite trajectories to less than 1 km error. However, this accuracy rapidly degenerates with time as the orbit is extrapolated into the future, beyond the time of the tracking data. Moreover, assuming that the data represents reasonably world-wide coverage, increasing the amount of data does not reduce the tracking error by the expected statistical factor. Experiences of this sort are becoming common to those associated with accurate satellite tracking systems and lead one to the conclusion that the principal factor currently limiting tracking accuracy are errors in parameters associated with the earth's gravitational force field and locations of the tracking stations. Consequently, if a significant reduction in tracking error is to be achieved, improved values for these geodetic parameters must be determined.

One of the chief contributors to accurate tracking systems has been the Applied Physics Laboratory, The Johns Hopkins University through development of the Navy's doppler tracking system.^{1,2} The principal objective of this tracking system is to supply the orbital parameters specifying the trajectories of the doppler instrumented satellites to an accuracy of the

order of 0.1 km when projected into the future for one or two days. This is a most severe accuracy requirement and considerable effort is being spent on determining improved values for the necessary geodetic parameters in order to eventually achieve this accuracy.³ This paper will be a brief summary of the main problems associated with the statistical inference of geodetic parameters, giving special attention to the use of radio doppler tracking data.

Significant improvements in many of the parameters associated with the earth's gravitation force field have already been made through studies of satellite motion.⁴⁻¹⁰ However, in most cases, they have been made without utilizing modern statistical techniques because the high sensitivity of the satellite motion to changes in these parameters obviates their need. The determination of the first parameter associated with the north-south asymmetry of the Earth - the so called "pear-shape" - is a good example of the methods that have been used so far.

Figure 1 indicates the change with time of the perigee altitude (distance of closest approach to the Earth) of the IB experimental satellite to orbit. To a first approximation, theory predicts that if the Earth were perfectly symmetrical between its northern and southern hemispheres, this perigee altitude would remain a constant. From Figure 1 it is clear that there must be an asymmetry between the hemispheres and O'Keefe in 1958 first determined that such a motion is caused principally by the Earth being shaped somewhat like a pear is shown in Figure 2.

It is interesting to note that the deviation of surfaces of constant gravitational potential (for example, the geoid) from north-south symmetry is less than 0.1 km while the amplitude of the variation in perigee altitude is about 15 km. This apparent amplification of a small deviation in the

Figure 1

1B ORBIT

PERIGEE RADIUS FROM CENTER OF EARTH

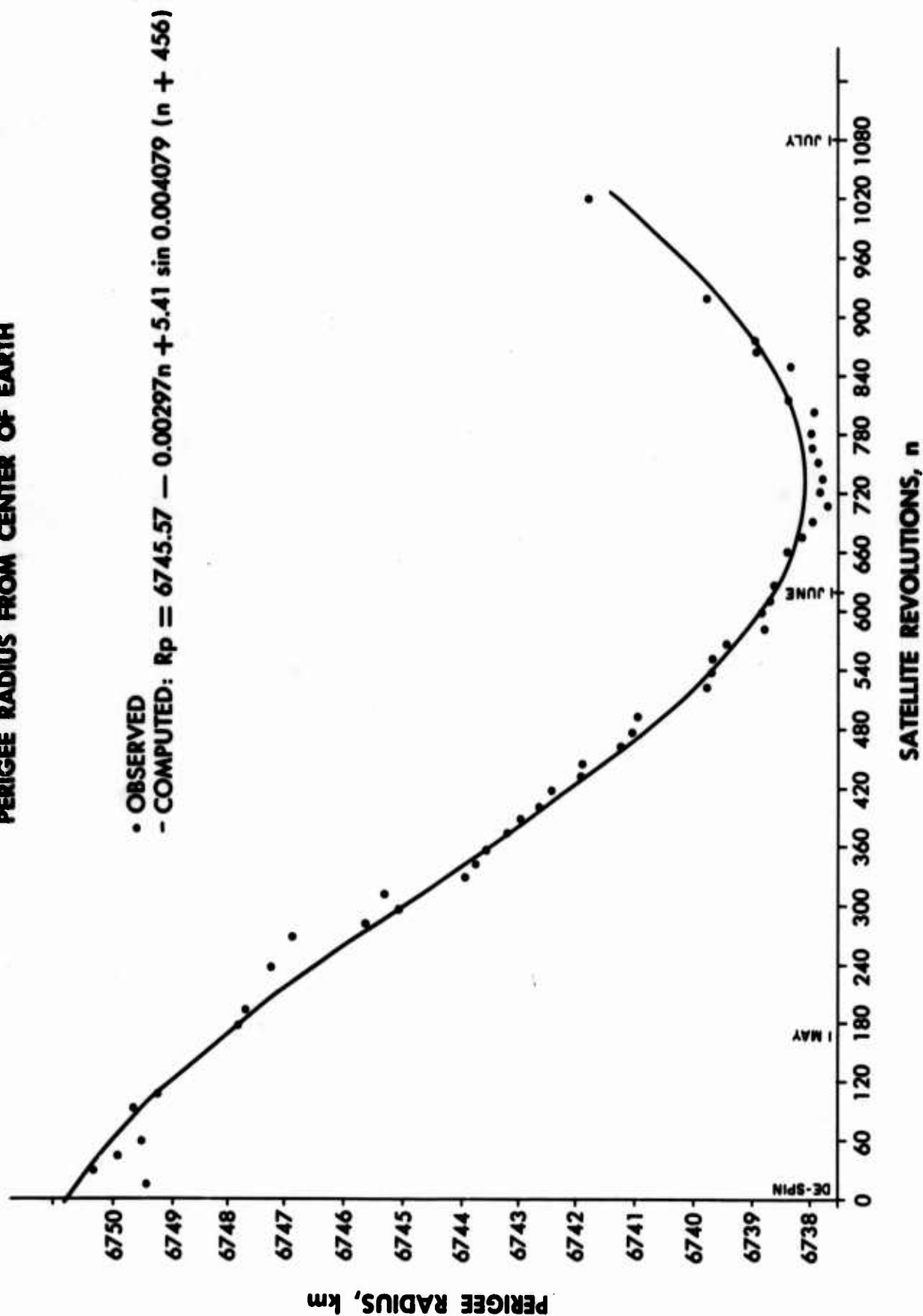
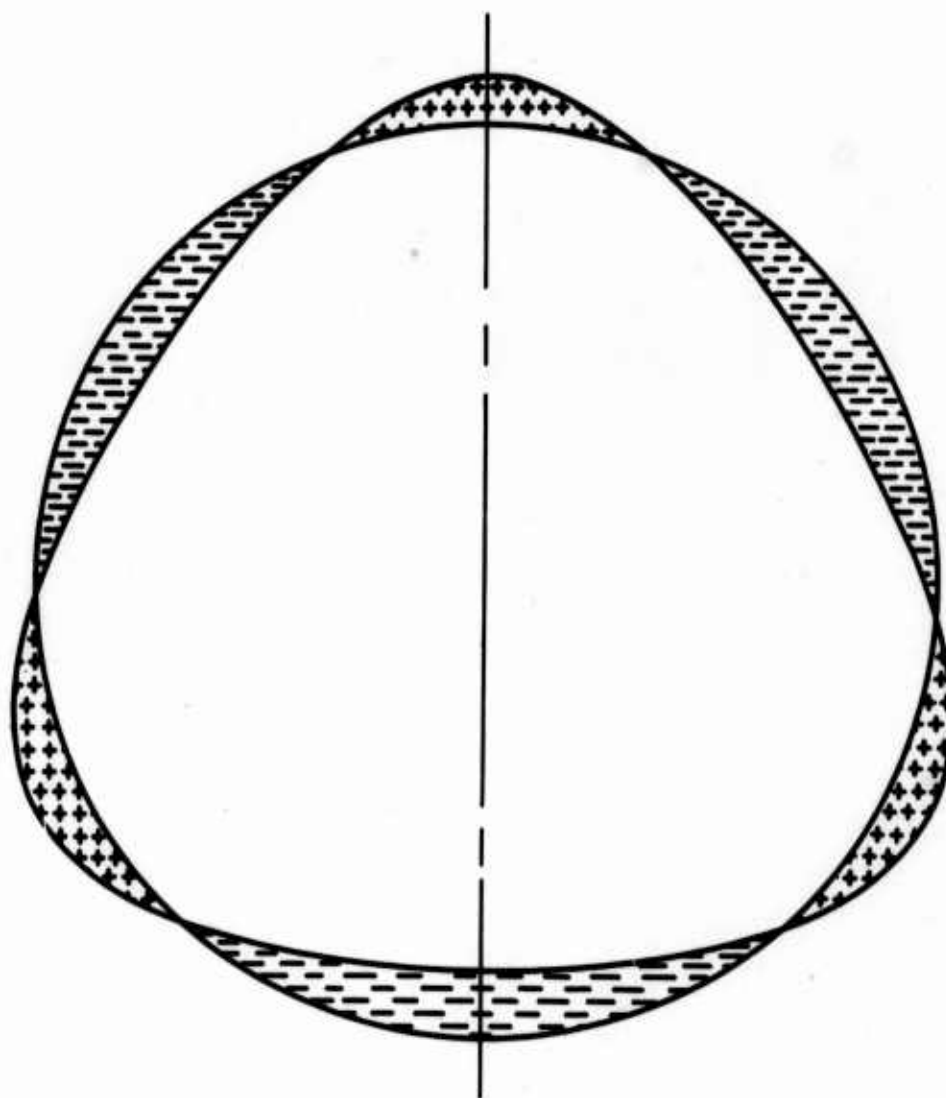


Figure 2
"PEAR-SHAPE" GRAVITATIONAL FORCE TERM



earth's geoid causing a large variation in the satellite motion because the satellite is experiencing a contribution to its force that keeps acting in the same way for many days so that a build-up to a relatively large value occurs. Several other force parameters cause a similar build-up of deviations in the satellite motion and consequently, relatively accurate values for the parameters can be determined without employing statistical techniques.

Figure 3 lists those parameters that have been determined to date, principally by the methods outlined above. In Figure 3, the coordinate r , represents the geocentric radius of the satellite from the earth's center of gravity, ϕ represents the geocentric latitude, and λ the geocentric longitude. The gravitational potential, $V(r, \phi, \lambda)$ has been expanded in the orthonormal spherical harmonics, $Y_{\ell}^{(m)}(\phi, \lambda)$, with expansion parameters $J_{\ell, m}$. Prior to the orbiting of earth satellites, only values for $J_{2,0}$ and $J_{4,0}$ had been estimated. In addition to providing estimates of the remaining parameters given in Figure 3, studies of satellite motion have materially improved the accuracy of $J_{2,0}$ and $J_{4,0}$. The most recent parameters to be determined are the $J_{2,2}$ and $J_{2,-2}$ associated with the Earth being rotationally asymmetric. Major contributors to the values of these parameters are given in the references listed in Figure 3. From Figure 3 it can be seen that the gravitational force field of the Earth can not be assumed to correspond to a simple flattened sphere but actually corresponds to a very complicated shape - one which probably will require 10 - 15 parameters to adequately describe it.

To date, little use has been made of satellite data to improve the location of tracking stations. One reason is that classical surveying methods have been developed to a high degree of accuracy throughout the years and the locations of tracking stations can be determined fairly accurately without the use of artificial satellites. In particular, survey methods can determine

Figure 3

PRINCIPAL GRAVITATIONAL FORCE PARAMETERS

$$V(r, \varphi, \lambda) = -\frac{K}{r} \left[1 + \sum_{l=2}^{\infty} \sum_{m=-l}^l \frac{J_{lm}}{r^l} Y_l^m(\varphi, \lambda) \right]$$

$J_{2,0}$ = 1st oblateness term^(4,5)

$J_{4,0}$ = 2nd oblateness term^(4,5,7)

$J_{3,0}$ = Pear shape term^(4,5,6)

$J_{5,0}; J_{7,0}$ = Higher order odd-harmonics^(4,5,6)

J_6 = Higher order even harmonics⁽⁷⁾

$J_{2,2} J_{2,-2}$ = 1st elliptic equator term^(8,9,10)

Rev DEC 1961

extremely accurately the location of one point relative to other near-by points. For example, relative locations of tracking stations within the continental United States are known to an accuracy that, until very recently, greatly exceeded the tracking accuracy of satellites. Another reason is that locating an isolated tracking station via tracking data depends upon the satellite trajectory being accurately known and, until many of the force parameters were reduced in error, the satellite trajectory could not be determined to an accuracy comparable to typical world-wide surveying accuracies.

However, there is a significant advantage to using satellite data to determine the locations of points on the Earth. This advantage is that when the analysis is properly performed the station coordinates are determined relative to the true center of mass and spin axis of the Earth - not just relative to another point assumed as a standard (datum point).^{11,12} Furthermore, with improvement in the accuracy of the force parameters, satellite data will provide very accurate geocentric station locations on a world-wide basis. In fact, by "doppler navigating" tracking stations there is clear evidence that the surveyed locations of some of the experimental doppler tracking stations are significantly in error.¹³

We are now ready to assess the magnitude of the task facing us in applying statistical inference techniques to determine accurately geodetic parameters from satellite tracking data. First, errors in satellite trajectories are strongly correlated with errors in the force field and, in turn, errors in the positions of tracking stations inferred from tracking data are correlated with errors in the satellite trajectories. Consequently, the three coordinates of each tracking station must be inferred along with the 10 - 15 force parameters.

Including the orbit parameters of the several satellites upwards of 60 parameters must be estimated. Clearly, one of the complicating factors from a practical standpoint is the very large number of statistically dependent parameters that must be inferred.

In addition, there are difficulties that are fundamental to the use of satellite data independent of the number of parameters that are to be inferred. Figure 4 lists the three principal ingredients that comprise our particular inference application and I shall now discuss some of the major difficulties contained in the first two items as they effect the third item.

The principal difficulty within item 1 of Figure 4 is that of including a sufficient number of the forces that act on the satellite. I have stated previously that there are probably 10 - 15 expansion parameters of the earth's gravitational potential that are significant. Actually, of course, there are many more parameters that could be included in this expansion and there are other forces that are not of a gravitational origin. However, it is clear that including their effect explicitly in the motion of the satellite becomes so difficult that a "cut-off" must be assumed and all force terms below this cut-off magnitude considered negligible. The remaining forces beyond this cut-off can now properly be termed "force noise" since, so far as their effect on the satellite motion is concerned, they are unknown.

Figure 5 lists some of the principal contributors to this force noise. Each of these forces is small in itself but, given sufficient time, its effect can build up to cause a significant error in the trajectory of the satellite. Consequently, any solution to the satellite equations of motion that does not explicitly include these forces, but which is adjusted continually to closely approximate the measured motion of the satellite over long periods of time,

Figure 4

**MAJOR FACTORS IN STATISTICAL INFERENCE
USING SATELLITE DATA**

1. Parameterization of the satellite motion in a convenient way to determine the unknown parameters.
2. Accurate statement of the statistical characteristics of the experimental data, including significant time correlations.
3. Development of an inference procedure which includes a sufficient number of the complicating factors present in the above items so that efficient use of the data is obtained.

Figure 5

SOURCES OF "FORCE NOISE"

1. Gravitational Forces
 - a. Gravity Anomalies, Mountain Ranges, etc.
 - b. Sea Tides
 - c. Neglected "low order" terms in the Gravitational Field
2. Non-Gravitational Forces
 - a. Air Drag Fluctuations
 - b. Electromagnetic Forces

yields a slow time dependence in the orbit parameters. This time dependence is analogous to a highly correlated random walk in the orbit parameter space. Estimates indicate that the orbit parameters will be significantly decorrelated after about one week of time. Consequently, either the statistical inference theory must include the ability of some of the parameters to vary randomly with time or data spanning time intervals no longer than about one week can be used in a single calculation.

The principal problems arising from item 2 of Figure 4 are the understanding and proper inclusion of the long correlation times that are present in the experimental tracking data. Figure 6 shows some of the major contributors to correlated errors in radio doppler data. The Navy's doppler receiving equipment is carefully designed with these sources in mind so that if no degradation in equipment performance in the field is experienced, station frequency and time errors should be small. For example, doppler instrumented satellites transmit timing signals which can be calibrated and checked with the Naval Observatory Time Standard. Each set of doppler data is assigned an independent frequency parameter to "fit-out" any errors in the station's frequency standard. However, in a world-wide tracking system it is not trivial to discover and totally eliminate subtle degradation and malfunctions of the station equipment and these can produce highly correlated errors in the data.

The second category of errors given in Figure 6 should also be small. The computing procedures used in tracking of doppler instrumented satellites parameterize the frequency of the satellite oscillator relative to the station's frequency and infer these parameters along with the satellite orbit parameters.^{2,11} Assuming that there is no malfunction in the doppler instrumented satellites, the satellite frequency changes very slowly with time so that frequency drift errors are reduced to a minimum. These frequency parameters are also used to

Figure 6

**SOURCES OF CORRELATED ERRORS
IN DOPPLER TRACKING DATA**

1. Tracking Station Equipment
 - a. Time and Frequency Errors
 - b. Unrecognized Equipment Mal-functions
2. Uncorrected Satellite Transmitter Frequency Drift
3. Uncorrected Ionospheric and Tropospheric Refraction

calibrate the satellite clock rate to insure the accurate calibration of the timing signals that the tracking stations receive from the satellites.

The third category of errors listed in Figure 6 are not so easily eliminated as the first two. An a priori tropospheric refraction correction can be made rather reliably if no unusual effects such as ducting are present. Moreover, multiple frequency data is taken by all of the doppler tracking stations to eliminate the frequency dependent ionospheric refraction effects.¹³ However, it is not possible to entirely eliminate refraction effects and they do introduce highly correlated errors.¹⁴

In summary, because of the existence of these sources of error, the characteristic of the noise components of the tracking data are highly correlated and difficult to specify. Continued studies are being made at the Applied Physics Laboratory to understand better their nature, reduce their magnitude to as low a value as possible, and develop methods for parameterizing their effects on the data.

It can now be seen that developing statistical inference procedures which will cope with the above difficulties is not a simple task. I will now briefly describe the general approach that is being adopted at the Applied Physics Laboratory and then conclude with a more careful consideration of the principal difficulty that is present in this approach. This approach is summarized in Figure 7.

First, we have established that current estimates of the locations of tracking stations from survey data are sufficiently accurate that the equations for the theoretical Doppler shift corresponding to the experimental doppler data can be linearized in the unknown parameters for station locations about their current values. Furthermore, it is believed that our knowledge of

Figure 7

GENERAL PROCEDURE FOR STATISTICAL INFERENCE

1. Linearize all equations in the unknown parameters and use a least squares, minimum variance procedure.
2. Numerically integrate the satellite equations of motion and the partial derivatives of the motion WRT the unknown parameters.
3. Determine new satellite orbit parameters periodically to account for the force noise.

the gravitation force field will improve to the point that soon the theoretical equations for the Doppler shift can be linearized in the unknown force parameters about their most recently estimated values. In addition, careful checks indicate that the errors in the experimental data, while highly correlated, are sufficiently close to being normally distributed with zero mean that a linear, least squares, minimum variance procedure should be adequate.

Second, no attempt is being made to describe the motion of the satellites via a perturbation theory because there are too many forces that must be included explicitly to make the motion tractable to a perturbation theory of sufficient accuracy. Consequently, all equations connected with the satellite motion are being numerically integrated. Also, it is eventually planned to numerically integrate the partial derivatives of the satellite position and velocity with respect to the force parameters. This reduces the problem of deriving a dependence of the theoretical expressions for the Doppler shift on the unknown parameters to one of computer programming.

Finally, as stated previously, estimates of the magnitude of the satellite force noise indicate that the satellite orbit parameters can be assumed to be constant to sufficient accuracy for time intervals up to about one week. Consequently, new experimental data over time intervals of about one week can be used in a single inference calculation to determine only time independent parameters. Each such inference calculation will yield improved values and a new variance-covariance matrix (actually its inverse) for the unknown parameters as a function of the second moments of the errors in the data, the values, and (the inverse of) the variance-covariance matrix that resulted from the previous calculation with a previous week's data. Consequently, except for the problem of implementing the calculations on a computer, the inference problem is reduced to deriving sufficiently accurate expressions for

the second moments of the errors in both the data and the unknown parameters which include the significant correlations and the force noise.

The principal difficulty in implementing such a minimum variance approach is the proper accounting of the correlations in the errors. When using any one week's tracking data to perform a small correction to all of the unknown parameters, the amount of the correction applied via a minimum variance estimate depends very strongly upon the estimated errors in the data and the variance-covariance matrix of the unknown parameters. With a high data rate system such as a doppler tracking system there is a strong tendency to neglect small but highly correlated errors to a catastrophic extent. I shall now, as my final topic, present the results of a simulation which was designed to study the effect of neglecting correlations in the expressions for the second moments of the errors.

The minimum variance estimate for the location of a single doppler receiving station was considered. A relatively simple example was taken by making the following assumptions. First, the errors in the doppler data used for the estimate were assumed to be highly correlated within a single pass of the satellite above the receiving station's horizon. Second, data taken during different satellite passes were assumed to be negligibly correlated. Thus, in effect, each pass of the satellite above the station's horizon was considered as the smallest set of data that yields independent statistics. Third, it was assumed that no correlation existed between errors in the doppler data from the receiving station and errors in the trajectory of the satellite. This assumption is equivalent to assuming that the receiving station's data was not used to determine the satellite trajectory. Fourth, it was assumed that the errors in the trajectory of the satellite were highly correlated from pass to pass to simulate the existence of errors in the forces acting on the satellite and errors in the locations of the tracking stations used to determine the orbit.

A correlation time of roughly 20 passes was assumed in the errors in the satellite trajectory to simulate the decorrelating effect of the relative geometry of the satellite and tracking stations changing markedly as the earth rotates under the satellite orbit. All errors were assumed to be normally distributed. The above assumptions are summarized in Figure 8.

The minimum variance technique employed was one where each set of doppler data was used to determine improved values for the three coordinates of the station along with a new variance-covariance matrix. The statistical weights employed for each set of new data were such as to minimize the new variance of the total station error.

Two cases were considered. In the first case the new variance-covariance matrix was computed including all of the above correlations in the errors, in particular the high degree of correlation in the satellite trajectory errors. In the second case, the correlations in the satellite trajectory errors were neglected so that, in effect, it was assumed that the satellite errors were uncorrelated from one satellite pass to the next. In both cases it was (arbitrarily) assumed that the probable error in the satellite trajectory was one-half of the total probable error in determining the station location from a single set of data. Also, in both cases, prior to the use of any doppler data it was assumed that the initial station error was infinite.

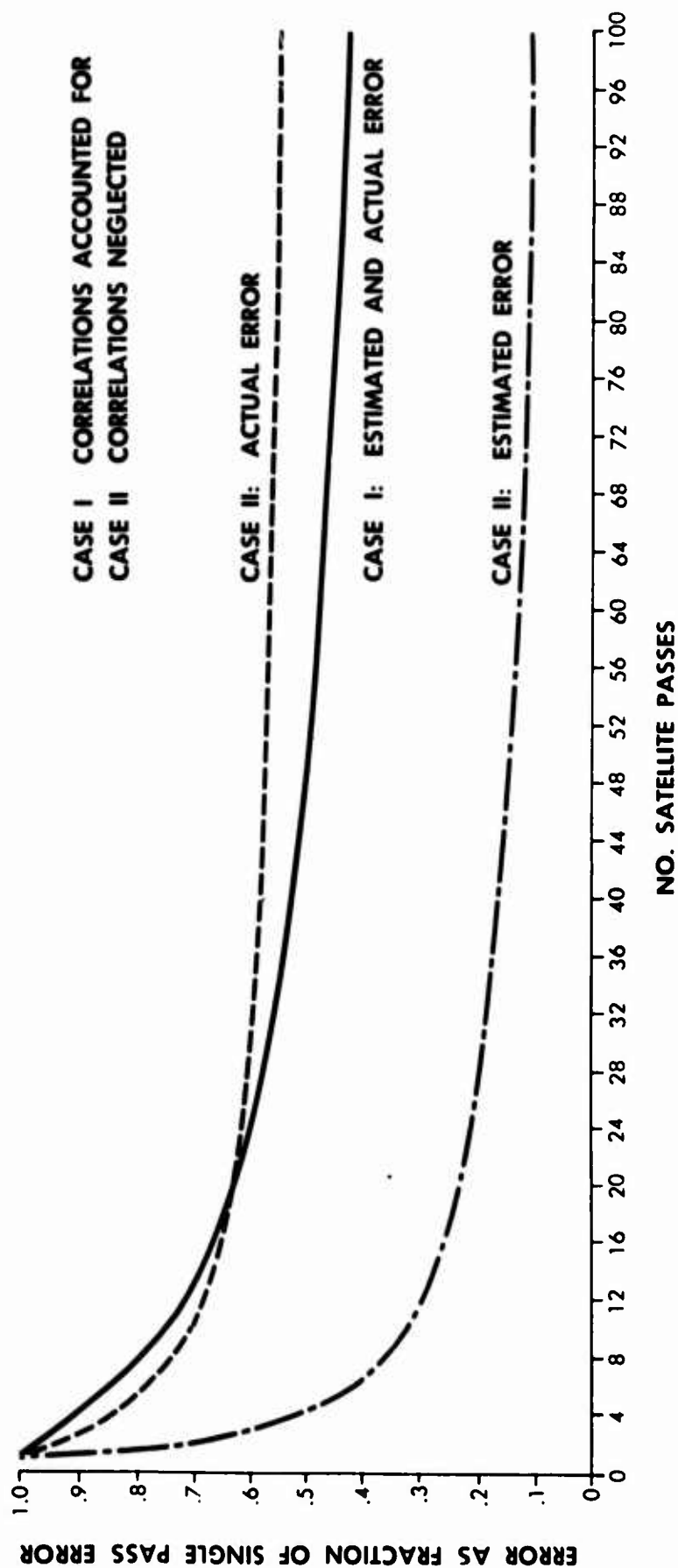
The results are shown in Figure 9. In Figure 9, the abscissa is the number of satellite passes used and the ordinate is the resulting station error stated as a fraction of the error in the station location resulting from using a single set of data. The results for case 1 are indicated by the solid line. For this case the estimated probable error and actual error are the same since all correlations were included in the minimum variance procedure. For case 2, the long dashed line indicates the estimated probable error that resulted

Figure 8

ASSUMPTIONS EMPLOYED IN SIMULATION

1. Errors in data points within a single pass are highly correlated.
2. Errors in data points between two different passes are uncorrelated.
3. Errors in data points are uncorrelated with satellite trajectory errors.
4. Satellite trajectory errors between adjacent passes are highly correlated.

Figure 9
SIMULATED MINIMUM VARIANCE ESTIMATE
OF
DOPPLER STATION LOCATION
EFFECT OF CORRELATIONS IN SATELLITE TRAJECTORY ERRORS



when the correlations in the satellite trajectory were neglected while the dotted line indicates the actual error resulting when the minimum variance procedure does not include the satellite trajectory correlations.

From Figure 9, it can be seen that when minimum variance inference procedures are used, and correlations in the errors are neglected, the probable error can be seriously underestimated. Even more serious, the actual error can be "frozen" at a larger value than occurs when the proper correlations are included in the estimating procedure. This 'freezing' of the corrections to the station's coordinates occurs because of the low estimate of the probable error that results from neglecting correlations.

From this example, it can be seen that it is very important to accurately estimate the characteristics of the errors that are inputs to a minimum variance estimating procedure. In fact, it appears that an overestimate of the probable error is less serious than an underestimate. To summarize, when minimum variance procedures are used with high data rate systems and when there are a large number of parameters to be inferred from the data, the effect of freezing the values of parameters because of underestimating their probable errors (as exemplified above) can introduce serious errors in the inferred values. Because of the complicated nature of the satellite motion, the correlations in the experimental data, and the large number of parameters that are to be estimated, this problem appears to require the most attention when inferring geodetic parameters from satellite tracking data.

Computer programs to implement a minimum variance estimate of the many parameters associated with the accurate tracking of doppler instrumented satellites are now under development at the Applied Physics Laboratory. The principal objective of these programs is to combine doppler and optical tracking

data from these satellites with existing high accuracy surface surveys to determine the gravitational force field of the Earth and the location of the doppler tracking stations to an accuracy compatible with the requirements to predict for approximately one day into the future the position of the doppler instrumented satellites within about 0.1 km error.* This development is proceeding along the lines discussed in this paper with particular emphasis being placed upon the development of special computer programs which will properly estimate the correlations in the errors present in the data and the variance-covariance matrix of the unknown parameters.

* Note added in proof.

The ANNA Geodetic Satellite is expected to contribute materially to the high quality doppler optical data continually being received from the experimental doppler instrumented satellites.

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